Modeling Ebola Outbreak: A Case Study on 2014 Outbreak in Sierra Leone

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ABSTRACT

In recent time Sierra Leone has gone through the deadliest Ebola outbreak. In this paper, we modified the well-known Susceptible-Exposed-Infectious-Removed (SEIR) model to determine the effect of vaccination and hospitalization of individuals to control the epidemic. Using the publicly available data for Sierra Leone provided by the WHO, our results show that vaccination and hospitalizing people can be significantly helpful to control the epidemic. We also discussed optimal control problem to determine the effect of putting optimal control on hospitalization.

Categories and Subject Descriptors

I.6.5 [**Simulation and Modeling**]: Model Development—*modeling methodologies*

General Terms

Measurement, Design, and Experimentation.

Keywords

Ebola, Mathematical modeling, SEIR model

1. INTRODUCTION

The mortality rate of the recent Ebola 2014 outbreak is reported to be around 71% as of September 2014 and the total number of reported cases exceed the combined number of patients from all the previously known outbreaks which makes it the most deadly Ebola outbreak ever. In this paper, we modified the Susceptible-Exposed-Infectious-Removed (SEIR) model, a widely used model for infectious diseases, to mathematically model the Ebola outbreak in Sierra Leone, the country where the maximum number of cases are reported. Using the modified SEIR model, we investigate the effect of vaccination and hospitalization of the infected individuals over time to control the epidemic. Putting an optimal control to hospitalization to end the epidemic is also discussed.

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2. METHODS

2.1 Datasets

A time series of reported Ebola cases in Sierra Leone is collected from public data released by the World Health Organization (WHO).

2.2 Mathematical Model

SEIR model [1] divides the population in four compartments, $\boldsymbol{S}$: Number of individuals who are susceptible to the disease, i.e. who are at risk of contracting the disease; $\boldsymbol{E}$: Number of individuals who are exposed, i.e. who are infected but are not capable of transmitting the disease; $\boldsymbol{I}$: Number of infected individuals who are capable of transmitting the disease; $\boldsymbol{R}$: Number of individuals who recovered from the disease with full immunity and have no chance of reinfection. This group also includes people who died from the disease. The SEIR model can be described using ordinary differential equation as follows:

$$\begin{aligned}\frac{dS(t)}{t} &= \frac{-\beta S(t)I(t)}{N}\\ \frac{dE(t)}{t} &= \frac{\beta S(t)I(t)}{N} - kE(t)\\ \frac{dI(t)}{t} &= kE(t) - \gamma I(t)\\ \frac{dR(t)}{t} &= \gamma I(t)\end{aligned} \tag{1}$$

In SEIR model, susceptible individuals (S) enter the exposed compartment (E) at the per-capita rate $\frac{\beta I}{N}$, where β is the transmission rate per person per day. N denotes the total effective population size and $\frac{I}{N}$ is the probability that a contact is made with an infectious individual. Exposed individuals undergo an average incubation period of $\frac{1}{k}$ days before progressing to the infectious compartment I. Infectious individuals move to the compartment R at the per-capita rate γ [1]. Note that the population (N) during study time t remains constant i.e. $N = S(t) + E(t) + I(t) + R(t)$.

2.3 Modeling the effect of vaccination

We simulate the effect of vaccination in practical Ebola situation by slightly modifying the SEIR model as given below:

$$\begin{aligned}\frac{dS(t)}{t} &= \frac{-\beta S(t)I(t)}{N} - \nu S(t)\\ \frac{dE(t)}{t} &= \frac{\beta S(t)I(t)}{N} - kE(t)\\ \frac{dI(t)}{t} &= kE(t) - \gamma I(t)\\ \frac{dR(t)}{t} &= \gamma I(t) + \nu S(t)\end{aligned} \tag{2}$$

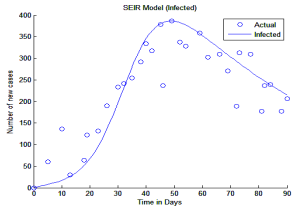

Figure 1: Result for fitting the model with actual data.

Here ν represents vaccination rate.

2.4 Modeling the effect of hospitalization

Similarly we simulate the effect of hospitalization. Here by the term "hospitalization", we mean immediately isolating the exposed patients from others and keep them in a place where there is no chance of spreading the disease. Moreover, we assume that in hospitals, there is such burial facility that it is not possible to get the disease spread from the dead body. Additionally, we also assume that people who take care of the patients in the hospitals take necessary precautions to protect themselves.

$$\begin{aligned}
\frac{dS(t)}{t} &= \frac{-\beta S(t)I(t)}{N} \\
\frac{dE(t)}{t} &= \frac{\beta S(t)I(t)}{N} - kE(t) - hE(t) \\
\frac{dI(t)}{t} &= kE(t) - \gamma I(t) \\
\frac{dR(t)}{t} &= \gamma I(t) + hE(t)
\end{aligned} \tag{3}$$

Here h represents hospitalization rate.

3. RESULTS

3.1 Model Fitting

To estimate the parameters of the original SEIR model (β, k, γ), we adapted the initialization of S, E, I, and R with the reported data by fitting the actual data of confirmed cases in Sierra Leone between August 3, 2014 and November 1, 2014. The result of fitting is shown in Figure 1. Out of the different values we tried, the mathematical model (1) fits well with the real data by using $\beta = 0.000318$ as the rate of transmission, $k = 0.15873$ as the incubation rate (average incubation period is 6.3 days) and $\gamma = 0.0175$ as the rate of recovery. The initial numbers of susceptible, exposed, infected, and recovered groups are given by $S(0) = 560$, $E(0) = 12$, $I(0) = 0$, and $R(0) = 0$, respectively.

3.2 Modeling the effect of vaccination and hospitalization

Different vaccination rates are applied to model (2) and the results are shown in Figure 2. We investigate the effect of vaccination particularly on the total number of recovered cases. For example, when the vaccination rate is 0% (Figure 2(a)), on day 50, total 146 cases are reported to be recovered. However, only 10% increase in the vaccination rate causes 3.5 times increase in the total number of recovered cases on day 50 (Figure 2(b)). Similarly there is an increase in the recovered cases when hospitalization rate is increased from 0% (Figure 3(a)) to 10% (Figure 3(b)).

4. DISCUSSION AND CONCLUSION

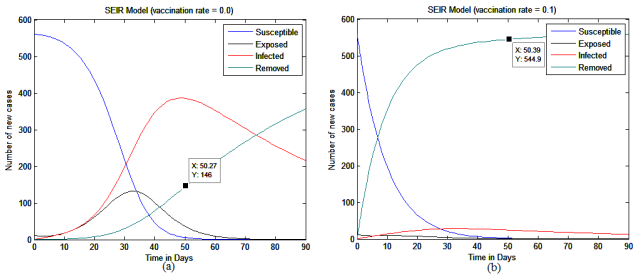

Figure 2: SEIR model with different rates of vaccination. (a) $\nu = 0\%$, (b) $\nu = 10\%$.

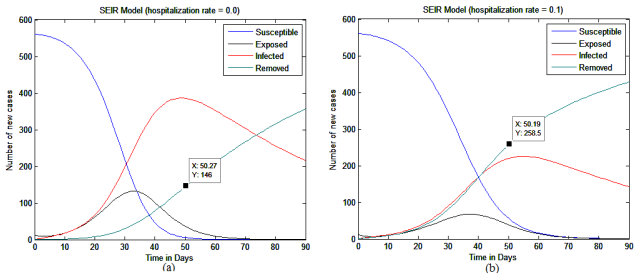

Figure 3: SEIR model with different rates of hospitalization. (a) $h = 0\%$, (b) $h = 10\%$.

From Figure 2 and 3 it is clear that vaccination is more effective than hospitalization. However, since no vaccination has been found so far for Ebola, we need to rely on hospitalizing more people. In best case, the hospitalization rate should be 100%, however, in reality it is not feasible. Therefore, we need to determine the optimal rate for hospitalization to keep the cost minimum while maximizing the number of recovered people. Therefore we present an optimal control problem by introducing a control to the hospitalization rate at time t. Let $\mu(t)$ represents the fraction of exposed individuals being hospitalized per unit of time. Then the corresponding differential equations can be written as:

$$\begin{aligned}
\frac{dS(t)}{t} &= \frac{-\beta S(t)I(t)}{N} \\
\frac{dE(t)}{t} &= \frac{\beta S(t)I(t)}{N} - kE(t) - \mu(t)E(t) \\
\frac{dI(t)}{t} &= kE(t) - \gamma I(t) \\
\frac{dR(t)}{t} &= \gamma I(t) + \mu(t)E(t)
\end{aligned} \tag{4}$$

Here the optimal control problem consists of minimizing the objective function.

$$f(\mu) = \int_0^T I(t) + \frac{A}{2}\mu^2(t)\,\mathrm{d}t \tag{5}$$

Subject to $0 \leq \mu(t) \leq 0.9$ for $t \in [0, T]$, $A \geq 0$ (A is the weight on cost and T is the duration of hospitalization). Using the control parameter, an optimized hospitalization rate can be determined where the cost of hospitalization can be minimized. Note that no numerical analysis is done as the data for hospitalization cost is not available to us.

5. REFERENCES

[1] G. Chowell, N. W. Hengartner, C. Castillo-Chavez, P. W. Fenimore, and J. M. Hyman. The basic reproductive number of Ebola and the effects of public health measures: the cases of Congo and Uganda. *Journal of Theoretical Biology*, 229(1):119–126, 2004.